Motion in a Plane

Question1

A bob is whirled in a horizontal circle by means of a string at an initial speed of 10rpm. If the tension in the string is quadrupled while keeping the radius constant, the new speed is:

[NEET 2024 Re]

Options:

A.

20 rpm

B.

40 rpm

C.

5 rpm

D.

10 rpm

Answer: A

Solution:

In horizontal circular motion,

$T = m\omega^2 r$

For constant m and r, $T \propto \omega^2$

$T' = 4T$ (Given)

$\Rightarrow \omega' = 2\omega$

$= 20$ rpm

Question2

Let ω_1, ω_2 and ω_3 be the angular speed of the second hand, minute hand and hour hand of a smoothly running analog clock, respectively. If x_1, x_2 and x_3 are their respective angular distances in 1 minute then the factor which remains constant(k) is

[NEET 2024 Re]

Options:

A.

$$\frac{\omega_1}{x_1} = \frac{\omega_2}{x_2} = \frac{\omega_3}{x_3} = k$$

B.

$$\omega_1 x_1 = \omega_2 x_2 = \omega_3 x_3 = k$$

C.

$$\omega_1 x_1^2 = \omega_2 x_2^2 = \omega_3 x_3^2 = k$$

D.

$$\omega_1^2 x_1 = \omega_2^2 x_2 = \omega_3^2 x_3 = k$$

Answer: A

Solution:

$$\omega_1 = \frac{2\pi}{60}; \quad x_1 = \frac{2\pi}{60} \times 60 = 2\pi$$

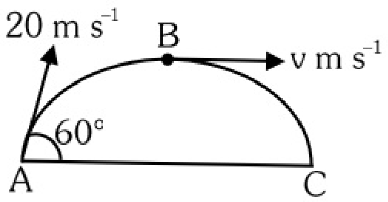
$$\omega_2 = \frac{2\pi}{3600}; \quad x_2 = \frac{2\pi}{3600} \times 60 = \frac{2\pi}{60}$$

$$\omega_3 = \frac{2\pi}{3600 \times 12}; \quad x_3 = \frac{2\pi}{3600 \times 12} \times 60 = \frac{2\pi}{720}$$

$$\frac{\omega_1}{x_1} = \frac{\omega_2}{x_2} = \frac{\omega_3}{x_3} = \frac{1}{60} = k$$

Question3

A ball is projected from point A with velocity 20m s^{-1} at an angle 60° to the horizontal direction. At the highest point B of the path (as shown in figure), the velocity vm s^{-1} of the ball will be:



[NEET 2023 mpr]

Options:

A.

20

B.

$10\sqrt{3}$

C.

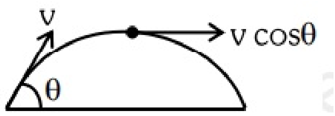
Zero

D.

10

Answer: D

Solution:



At the top most point of its trajectory particle will have only horizontal component of velocity

V at top = v cosθ

= 20 × 1/2

= 10m/ s

Question4

A particle is executing uniform circular motion with velocity $\vec{v}$ and acceleration $\vec{a}$. Which of the following is true?

[NEET 2023 mpr]

Options:

A.

$\vec{v}$ is a constant; $\vec{a}$ is not a constant

B.

$\vec{v}$ is not a constant; $\vec{a}$ is not a constant

C.

$\vec{v}$ is a constant; $\vec{a}$ is a constant

D.

$\vec{v}$ is not a constant; $\vec{a}$ is a constant

Answer: B

Solution:

Direction of velocity and centripetal acceleration changes continuously so $\vec{v}$ is not constant and $\vec{a}$ is not a constant.

Question5

A bullet is fired from a gun at the speed of 280m s^{-1} in the direction 30° above the horizontal. The maximum height attained by the bullet is (g = 9.8m s^{-2}, sin30° = 0.5)

[NEET 2023]

Options:

A.

2000m

B.

1000m

C.

3000m

D.

2800m

Answer: B

Solution:

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$H = \frac{(280)^2(\sin^2 30)}{2 \times 9.8}$$

$$= \frac{280 \times 280 \times 0.5 \times 0.5}{2 \times 9.8}$$

$$H = 1000\text{m}$$

Question6

A ball is projected with a velocity, $10ms^{-1}$, at an angle of 60° with the vertical direction. Its speed at the highest point of its trajectory will be [NEET-2022]

Options:

A. Zero

B. $5\sqrt{3}ms^{-1}$

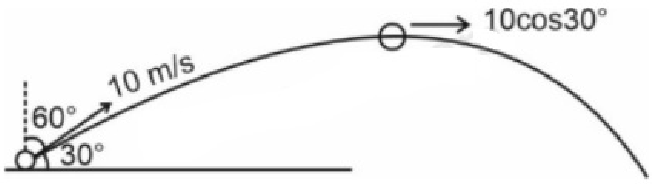
C. $5ms^{-1}$

D. $10ms^{-1}$

Answer: B

Solution:

At highest point vertical component of velocity become zero.



At highest point speed of object $= 10\cos 30^{\circ}$

$= 5\sqrt{3}m/s$

Question7

A cricket ball is thrown by a player at a speed of 20m / s in a direction 30° above the horizontal. The maximum height attained by the ball during its motion is.
$(g = 10m/s^2)$
[NEET Re-2022]

Options:

A. 25m

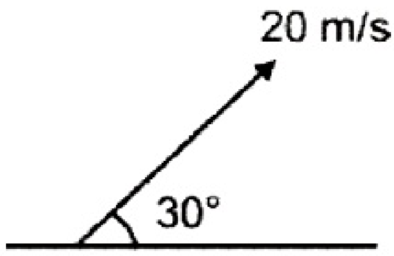
B. 5m

C. 10m

D. 20m

Answer: B

Solution:



$$H = \frac{u^2\sin^2\theta}{2g}$$

$= 5m$

Question8

If $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k}$, then the scalar and vector products of $\vec{F}$ and $\vec{r}$ have the magnitudes respectively as:
[NEET Re-2022]

Options:

A. 10,2

B. 5, $\sqrt{3}$

C. 4, $\sqrt{5}$

D. 10, $\sqrt{2}$

Answer: D

Solution:

Given $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}, \quad \vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

Scalar product $= \vec{F} \cdot \vec{r}$

$= 2 \times 3 + 1 \times 2 + (-1) \times (-2)$

$= 6 + 2 + 2 = 10$

Vector product $= \vec{F} \times \vec{r}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \hat{i}(-2+2) - \hat{j}(-4+3) + \hat{k}(4-3)$$

$\vec{F} \times \vec{r} = \hat{j} + \hat{k}$

$|\vec{F} \times \vec{r}| = \sqrt{2}$

Question9

A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution. If this particle were projected with the same speed at an angle ' θ ' to the horizontal, the maximum height attained by it equals 4R. The angle of projection, θ, is then given by :
[NEET 2021]

Options:

A. $\theta = \cos^{-1}\left(\frac{gT^2}{\pi^2R}\right)^{1/2}$

B. $\theta = \cos^{-1}\left(\frac{\pi^2R}{gT^2}\right)^{1/2}$

C. $\theta = \sin^{-1}\left(\frac{\pi^2R}{gT^2}\right)^{1/2}$

D. $\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2R}\right)^{1/2}$

Answer: D

Solution:

Solution:
. To complete a circular path of radius R, time period is T.
so speed of particle $(U) = \frac{2\pi R}{T}$ ....(1)
Now the particle is projected with same speed at angle θ to horizontal.
So Maximum Height $(H) = \frac{U^2\sin^2\theta}{2g}$
Given that : $H = 4R$
$\Rightarrow \frac{U^2\sin^2\theta}{2g} = 4R$
$\Rightarrow \sin^2\theta = \frac{8gR}{U^2}$ ....(2)
$\Rightarrow \sin^2\theta = \frac{8gRT^2}{4\pi^2R^2} = \frac{2gT^2}{\pi^2R}$ (using equation 1)
$\Rightarrow \theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2R}\right)^{1/2}$

Question10

The speed of a swimmer in still water is 20m / s. The speed of river water is 10m / s and is flowing due east.
If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is, given by
(NEET 2019)

Options:

A. 45° west

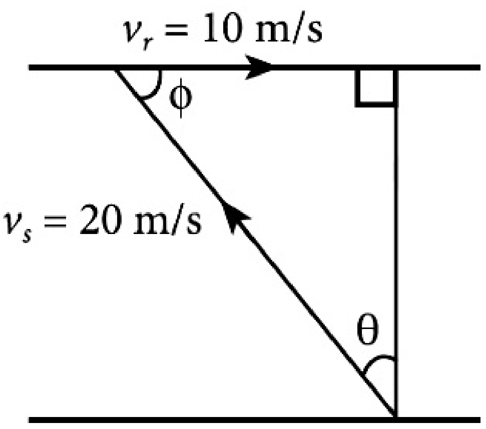
B. 30° west

C. 0°

D. 60° west

Answer: B

Solution:



$\cos\varphi = \frac{10}{20} = \frac{1}{2}$ or $\varphi = 60^\circ$
$\Rightarrow\ \theta = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

Question11

When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be
(NEET 2019)

Options:

A. $1 : 2\sqrt{3}$

B. $1 : \sqrt{2}$

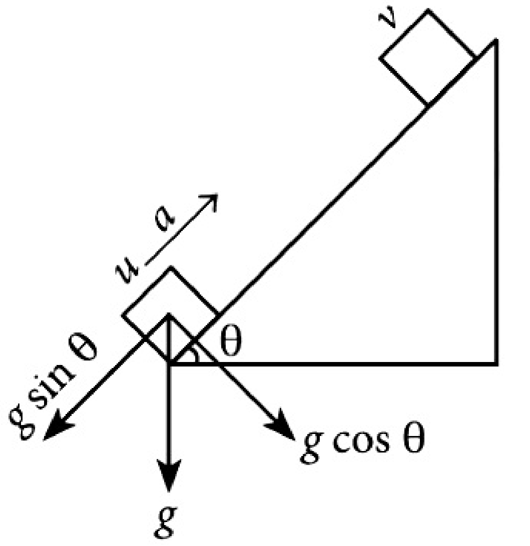
C. $\sqrt{2} : 1$

D. $1 : \sqrt{3}$

Answer: D

Solution:

$v^2 - u^2 = 2ax$
For case : I $v^2 = u^2 - 2(g\sin\theta_1)x_1$
$[\because a = -g\sin\theta]$



$x_1 = \frac{u^2}{2g\sin\theta_1}$ $[\because v = 0]$

For case II :

$v^2 = u^2 - (2g\sin\theta_2)x_2$

$x_2 = \frac{u^2}{2g\sin\theta_2} \therefore \frac{x_1}{x_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}}$

Question12

Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be
(NEET 2019)

Options:

A. 1: 1

B. $r_A : r_B$

C. $v_A : v_B$

D. $r_B : r_A$

Answer: A

Solution:

Time period, $T = \frac{2\pi}{\omega}$

As $T_A = T_B$

So, $\frac{2\pi}{\omega_A} = \frac{2\pi}{\omega_B}$ or $\omega_A : \omega_B = 1 : 1$

Question13

A particle starting from rest, moves in a circle of radius 'r. It attains a velocity of V_0 m / s in the n^{th} round. Its angular acceleration will be (OD NEET 2019)

Options:

A. $\frac{V_0}{n}$ rad / s^2

B. $\frac{V_0}{2\pi nr^2}$ rad / s^2

C. $\frac{V_0^2}{4\pi nr^2}$ rad / s^2

D. $\frac{V_0^2}{4\pi nr}$ rad / s^2

Answer: C

Solution:

Distance travelled in n^{th} rounds $= (2\pi r)n$

Using $v^2 = u^2 + 2as$ or, $V_0^2 = 0 + 2a(2\pi r)n$

$$a = \frac{V_0^2}{4\pi nr}$$

Angular acceleration, $\alpha = \frac{a}{r} = \frac{V_0^2}{4\pi nr^2}$ rad / s^2

Question14

The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at t = 2 s (2017 NEET)

Options:

A. $5 m / s^2$

B. $-4 m / s^2$

C. $-8 m / s^2$

D. 0

Answer: B

Solution:

$x = 5t - 2t^2$, $y = 10t$

$\frac{dx}{dt} = 5 - 4t$, $\frac{dy}{dx} = 10$

$\therefore v_x = 5 - 4t$, $\frac{dy}{dt} = 10$

$\frac{dv_x}{dt} = -4$, $\frac{dv_y}{dt} = 0$

$\therefore a_x = -4$, $a_y = 0$

Acceleration, $\vec{a} = a_x\hat{i} + a_y\hat{j} = -4\hat{i}$

$\therefore$The acceleration of the particle at $t = 2s$ is $-4ms^2$

Question15

If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is (2016 NEET Phase-I)

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Options:

A. 45

B. 180°

C. 0

D. 90

Answer: D

Solution:

Solution:

Let the two vectors be $\vec{A}$ and $\vec{B}$.

Then, magnitude of sum of $\vec{A}$ and $\vec{B}$

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

and magnitude of difference of $\vec{A}$ and $\vec{B}$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$,

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ (given)

or $\sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$\Rightarrow 4AB\cos\theta = 0$

$\because 4AB \neq 0$, $\therefore \cos\theta = 0$ or $\theta = 90°$

Question16

A particle moves so that its position vector is given by

$\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$,**where ω is a constant. which of the following is true ?**
(2016 NEET Phase-1)

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Options:

A. Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.

B. Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

C. Velocity and acceleration both are perpendicular to $\vec{r}$

D. Velocity and acceleration both are parallel to $\vec{r}$

Answer: A

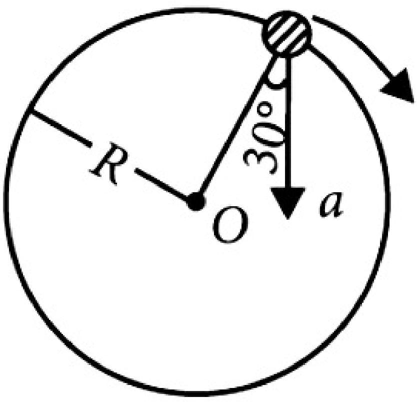
Solution:

Solution:
Given, $\vec{r} = \cos\omega t\hat{x} + \sin\omega t\hat{y}$
$\therefore \vec{v} = \frac{d\vec{r}}{dt} = -\omega\sin\omega t\hat{x} + \omega\cos\omega t\hat{y}$
$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2\cos\omega t\hat{x} - \omega^2\sin\omega t\hat{y} = -\omega^2\vec{r}$
Since position vector $(\vec{r})$
is directed away from the origin,so,acceleration $(-\omega^2\vec{r})$ is directed towards the origin.
Also $\vec{r}\cdot\vec{v}$
$= (\cos\omega t\hat{x} + \sin\omega t\hat{y})\cdot(-\omega\sin\omega t\hat{x} + \omega\cos\omega t\hat{y})$
$= -\omega\sin\omega t\cos\omega t + \omega\sin\omega t\cos\omega t = 0$

$\Rightarrow \vec{r} \perp \vec{v}$

Question17

In the given figure, a = 15ms^{-2} represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is



(2016 NEET Phase-II)

Options:

A. $4.5ms^{-1}$

B. $5.0ms^{-1}$

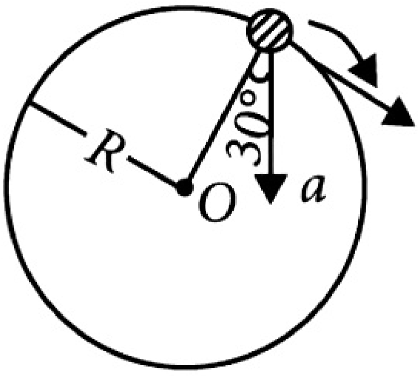
C. $5.7ms^{-1}$

D. $6.2ms^{-1}$

Answer: C

Solution:

Solution:



Here, $a = 15ms^{-2}$
$R = 2.5m$
From figure
$a_c = a\cos30°$
$= 15 \times \frac{\sqrt{3}}{2} ms^{-2}$
As we know, $a_c = \frac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$
$\therefore v = \sqrt{15 \times \frac{\sqrt{3}}{2} \times 2.5} = 5.96 \simeq 5.7ms^{-1}$

Question18

If vectors $\vec{A} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ and $\vec{B} = \cos\frac{\omega t}{2}\hat{i} + \sin\omega t\,\hat{j}$ are functions of time, then the value of t at which they are orthogonal to each other is (2015)

Options:

A. $t = \frac{\pi}{\omega}$

B. $t = 0$

C. $t = \frac{\pi}{4\omega}$

D. $t = \frac{\pi}{2\omega}$

Answer: A

Solution:

Solution:
Two vectors $\vec{A}$ and $\vec{B}$ are orthogonal to each other,if their scalar product is zero i . e . $\vec{A} \cdot \vec{B} = 0$
Here $\vec{A} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$
and $\vec{B} = \cos\frac{\omega t}{2}\hat{i} + \sin\omega t\hat{j}$
$\therefore \vec{A} \cdot \vec{B} = \left(\cos\omega t\hat{i} + \sin\omega t\hat{j}\right) \cdot \left(\cos\frac{\omega t}{2}\hat{i} + \sin\frac{\omega t}{2}\hat{j}\right)$

$= \cos\omega t\cos\frac{\omega t}{2} + \sin\omega t\sin\frac{\omega t}{2}$
$\left(\because \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = 1 \text{ and } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0\right)$

$= \cos\left(\omega t - \frac{\omega t}{2}\right)$
$(\because \cos(A - B) = \cos A\cos B + \sin A\sin B)$
But $\vec{A} . \vec{B} = 0 \left(\text{as} \vec{A} \text{and } \vec{B}\right)$ are orthogonal to each other
$\therefore \cos\left(\omega t - \frac{\omega t}{2}\right) = 0$
$\cos\left(\omega t - \frac{\omega t}{2}\right) = \cos\frac{\pi}{2}$ or $\omega t - \frac{\omega t}{2} = \frac{\pi}{2}$
$\frac{\omega t}{2} = \pi$ or $t = \frac{\pi}{\omega}$

Question19

The position vector of a particle $\vec{R}$ as a function of time is given by
$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$
Where R is in meters,t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x – and y – directions , respectively. Which one of the following statements is wrong for the motion of particle?
(2015)

Options:

A. Magnitude of the velocity of particle is 8 metre/second.

B. Path of the particle is a circle of radius 4 metre.

C. Acceleration vector is along $-\vec{R}$

D. Magnitude o f acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle.

Answer: A

Solution:

Here $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$
The velocity of the particle is

$\vec{v} = \frac{d\vec{R}}{dt} = \frac{d}{dt}\left[4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}\right]$

$= 8\pi\cos(2\pi t)\hat{i} - 8\pi\sin(2\pi t)\hat{j}$

Its magnitude is

$|\vec{v}| = \sqrt{(8\pi\cos(2\pi t))^2 + (-8\pi\sin(2\pi t))^2}$

$= \sqrt{64\pi^2\cos^2(2\pi t) + 64\pi^2\sin^2(2\pi t)}$

$= \sqrt{64\pi^2[\cos^2(2\pi t) + \sin^2(2\pi t)]}$

$= \sqrt{64\pi^2}$ (as $\sin^2\theta + \cos^2\theta = 1$)

$= 8\pi$m/s

Question20

A ship A is moving westwards with a speed of $10 kmh^{-1}$ and a ship B 100 km South of A, is moving northwards with a speed of $10 kmh^{-1}$.The time after which the distance between them becomes shortest, is (2015)

Options:

A. $5\sqrt{2}$h

B. $10\sqrt{2}$h

C. 0 h

D. 5 h

Answer: D

Solution:

Solution:

: Given situation is shown in the figure

OS = 100 km

OP = Shortest distance

Relative velocity between A and B is

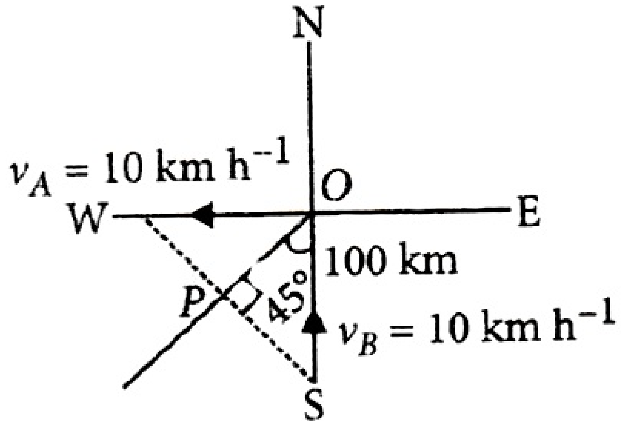
$v_{AB} = \sqrt{v_A^2 + v_B^2} = 10\sqrt{2} kmh^{-1}$

$\cos 45° = \frac{OP}{OS}, \frac{1}{\sqrt{2}} = \frac{OP}{100}$

$OP = \frac{100}{\sqrt{2}} = \frac{100\sqrt{2}}{2} = 50\sqrt{2}$km

The time after which distance between them equals to OP is given by

$t = \frac{OP}{v_{AB}} = \frac{50\sqrt{2}}{10\sqrt{2}} \Rightarrow t = 5h$



Question21

A projectile is fired from the surface of the earth with a velocity of $5ms^{-1}$ and angle θ with horizontal. Another projectile fired from another planet with a velocity of $3ms^{-1}$ at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is
(Given: $g = 9.8ms^{-2}$)
(2014)

Options:

A. 3.5

B. 5.9

C. 16.3

D. 110.8

Answer: A

Solution:

The equation of trajectory is
$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$
where 0 is the angle of projection and u is the velocity
For equal trajectories for same angles of projection, $\frac{g}{u^2} = \text{constant}$
As per question, $\frac{9.8}{5^2} = \frac{g'}{3^2}$
Where g' is acceleration due to gravity on the planet
$g' = \frac{9.8 \times 9}{25} = 3.5ms^{-2}$

Question22

A particle is moving such that its position coordinates (x, y) are (2 m, 3 m) at time t = 0, (6 m, 7 m) at time t = 2 s and (13 m, 14 m) at time t = 5 s.
Average velocity vector $(\vec{v}_{av})$ from t = 0 to t = 5 s is
(2014)

Options:

A. $\frac{1}{5}\left(13\hat{i} + 14\hat{j}\right)$

B. $\frac{7}{3}\left(\hat{i} + \hat{j}\right)$

C. $2\left(\hat{i} + \hat{j}\right)$

D. $\frac{11}{5}\left(\hat{i} + \hat{j}\right)$

Answer: D

Solution:

Solution:

At time t = 0, the position vector of the particle is $\vec{r}_1 = 2\hat{i} + 3\hat{j}$

At time t=5 s,the position vector of the particle is

$\vec{r}_2 = 13\hat{i} + 14\hat{j}$

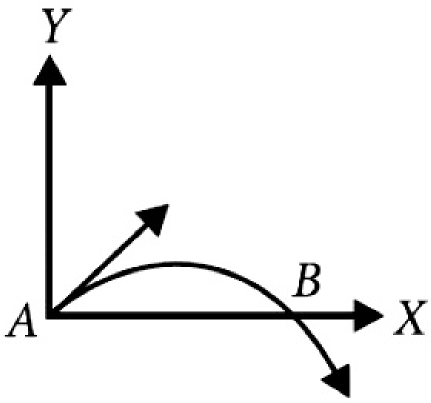
Displacement from $\vec{r}_1$ to $\vec{r}_2$ is

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left(13\hat{i} + 14\hat{j}\right) - \left(2\hat{i} + 3\hat{j}\right) = 11\hat{i} + 11\hat{j}$

∴ Average velocity, $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{11\hat{i} + 11\hat{j}}{5 - 0} = \frac{11}{5}\left(\hat{i} + \hat{j}\right)$

Question23

The velocity of a projectile at the initial point A is $\left(2\hat{i} + 3\hat{j}\right)$ m/s. It's velocity (in m/s) at point B is



(2013 NEET)

Options:

A. $2\hat{i} - 3\hat{j}$

B. $2\hat{i} + 3\hat{j}$

C. $-2\hat{i} - 3\hat{j}$

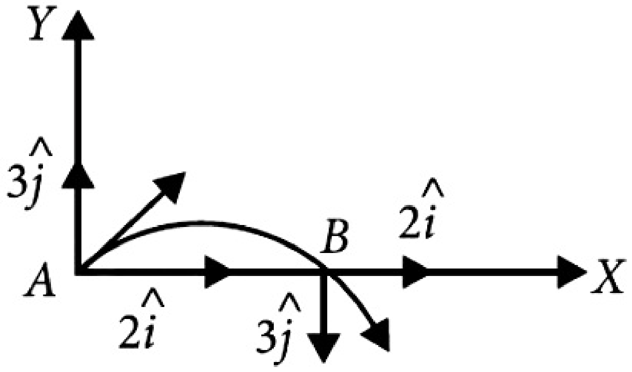
D. $-2\hat{i} + 3\hat{j}$

Answer: A

Solution:

Solution:
At point B, X component of velocity remains unchanged while Y component reverses its direction.

∴ The velocity of the projectile at point B is $2\hat{i} - 3\hat{j}$ m/s



Question24

Vectors, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$. Then the vector parallel to $\vec{A}$ is
(KN NEET 2013)

Options:

A. $\vec{A} \times \vec{B}$

B. $\vec{B} + \vec{C}$

C. $\vec{B} \times \vec{C}$

D. $\vec{B}$ and $\vec{C}$

Answer: C

Solution:

(c) : Vector triple product of three vectors
$\vec{A}$, $\vec{B}$ and $\vec{C}$ is $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$
Given : $\vec{A} \cdot \vec{B} = 0$, $\vec{A} \cdot \vec{C} = 0$
∴ $\vec{A} \times (\vec{B} \times \vec{C}) = 0$
Thus the vector $\vec{A}$ is parallel to vector $\vec{B} \times \vec{C}$.

Question25

The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is
(2012)

Options:

A. $\theta = \tan^{-1}\left(\frac{1}{4}\right)$

B. $\theta = \tan^{-1}(4)$

C. $\theta = \tan^{-1}(2)$

D. $\theta = 45°$

Answer: B

Solution:

Solution:

Horizontal range,H $= \frac{u^2\sin^2\theta}{2g}$

According to question $R = H$

$\therefore \frac{u^2\sin2\theta}{g} = \frac{u^2\sin^2\theta}{2g}, \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin^2\theta}{2g}$

$\tan\theta = 4$ or $\theta = \tan^{-1}(4)$

Question26

A particle has initial velocity $\left(2\vec{i} + 3\vec{j}\right)$ and acceleration $\left(0.3\vec{i} + 0.2\vec{j}\right)$. The magnitude of velocity after 10 seconds will be
(2012)

Options:

A. $9\sqrt{2}$ units

B. $5\sqrt{2}$ units

C. 5 units

D. 9 units

Answer: B

Solution:

Here, $\vec{u} = 2\hat{i} + 3\hat{j}$, $\vec{a} = 0.3\hat{i} + 0.2\hat{j}$, $t = 10s$
As $\vec{v} = \vec{u} + \vec{a}t$

$\therefore \vec{v} = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j})(10)$
$= 2\hat{i} + 3\hat{j} + 3\hat{i} + 2\hat{j} = 5\hat{i} + 5\hat{j}$
$|\vec{v}| = \sqrt{(5)^2 + (5)^2} = 5\sqrt{2}$ units

Question27

A particle moves in a circle of radius 5 cm with constant speed and time period 0.2πs . The acceleration of the particle is (2011)

Options:

A. $15\frac{m}{s^2}$

B. $25\frac{m}{s^2}$

C. $36\frac{m}{s^2}$

D. $5\frac{m}{s^2}$

Answer: D

Solution:

Solution:
Here, Radius, $R = 5cm = 5 \times 10^{-2}m$
Time period, $T = 0.2\pi s$
Centripetal acceleration
$a_c = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5 \times 10^{-2}) = 5\frac{m}{s^2}$

As particle moves with constant speed, therefore its tangential acceleration is zero. So, $a_t = 0$
The acceleration of the particle is
$a = \sqrt{a_c^2 + a_t^2} = a_c = 5\frac{m}{s^2}$
It acts towards the centre of the circle

Question28

A missile is fired for maximum range with an initial velocity of 20 m/s. If $g = 10\frac{m}{s^2}$, the range of missile is (2011)

Options:

A. 40 m

B. 50 m

C. 60 m

D. 20 m

Answer: A

Solution:

Solution:

Here, $u = 20\frac{m}{s}$, $g = 10\frac{m}{s^2}$

For maximum range, angle of projection is $\theta = 45°$

$\therefore R_{max} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \quad \left(\because R = \frac{u^2 \sin 2\theta}{g}\right)$

$$= \frac{\left(20\frac{m}{s}\right)^2}{\left(10\frac{m}{s^2}\right)} = 40m$$

Question29

A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is (2011)

Options:

A. $1\frac{m}{s^2}$

B. $7\frac{m}{s^2}$

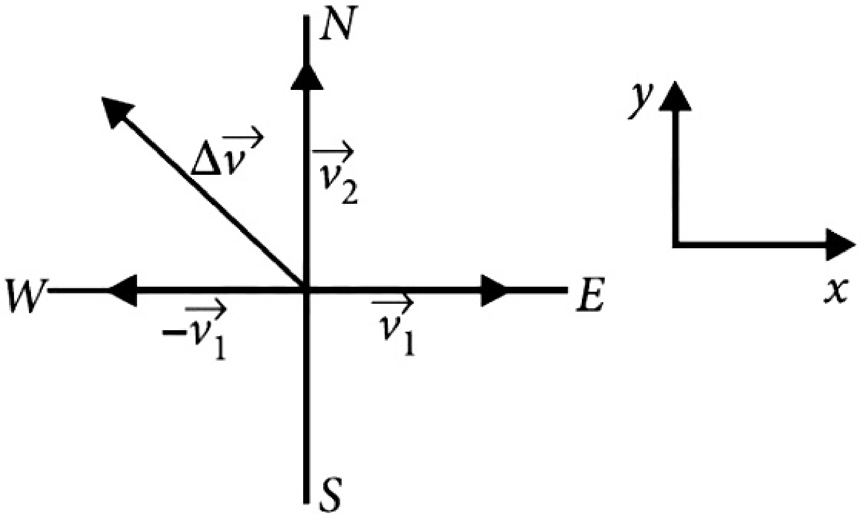
C. $\sqrt{7}\frac{m}{s^2}$

D. $5\frac{m}{s^2}$

Answer: D

Solution:

Solution:



Velocity towards east direction, $\vec{v}_1 = 30\hat{i}$ m/s

Velocity towards north direction, $\vec{v}_2 = 40\hat{j}$ m/s

change in velocity, $\Delta v = \vec{c}_2 - \vec{v}_1 = \left(40\hat{j} - 30\hat{i}\right)$

$\therefore \left|\Delta\vec{v}\right| = \left|40\hat{j} - 30\hat{i}\right| = 50$ m/s

Average acceleration, $\vec{a}_{av} = \frac{\text{Change in velocity}}{\text{Time interval}}$

$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$

$\left|\vec{a}_{av}\right| = \frac{\left|\Delta\vec{v}\right|}{\Delta t} = \frac{50\text{m / s}}{10\text{s}} = 5\text{m / s}^2$

Question30

A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is
(2011)

Options:

A. 45°

B. 60

C. $\tan^{-1}\left(\frac{1}{2}\right)$

D. $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer: C

Solution:

Solution:
Let ϕ be elevation angle of the projectile at its highest point as seen from the point of projection O and θ be angle of projection with the horizontal. From figure,

$\tan\phi = \frac{H}{R/2}$ ....(i)

In case of projectile motion

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

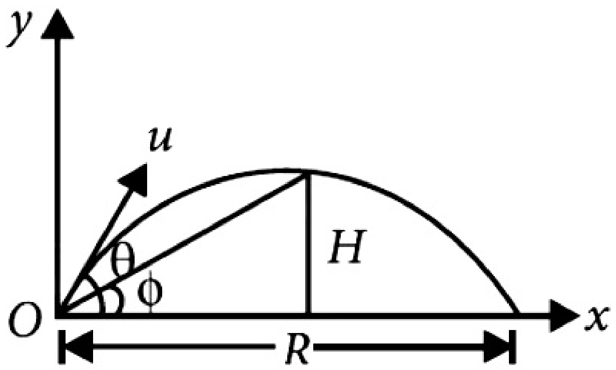
Horizontal range, $R = \frac{u^2\sin2\theta}{g}$

Substituting these values of H and R in (i), we get

$\tan\phi = \frac{\sin^2\theta}{\sin 2\theta} = \frac{\sin^2\theta}{2\sin\theta\cos\theta} = \frac{1}{2}\tan\theta$

Here, $\theta = 45°$

$\therefore \tan\phi = \frac{1}{2}\tan 45° = \frac{1}{2}; \ \phi = \tan^{-1}\left(\frac{1}{2}\right)$



Question31

A particle has initial velocity $(3\hat{i} + 4\hat{j})$ and has acceleration $(0.4\hat{i} + 0.3\hat{j})$. Its speed after 10 s is (2010)

Options:

A. 7 units

B. $7\sqrt{2}$ units

C. 8.5 units

D. 10 units

Answer: B

Solution:

Here,

Initial velocity, $\vec{u} = 3\hat{i} + 4\hat{j}$

Acceleration, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$

Time, t = 10 s

Let $\vec{v}$ be velocity of a particle after 10s.

Using $\vec{v} = \vec{u} + \vec{a}t$

$\therefore \vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})(10)$

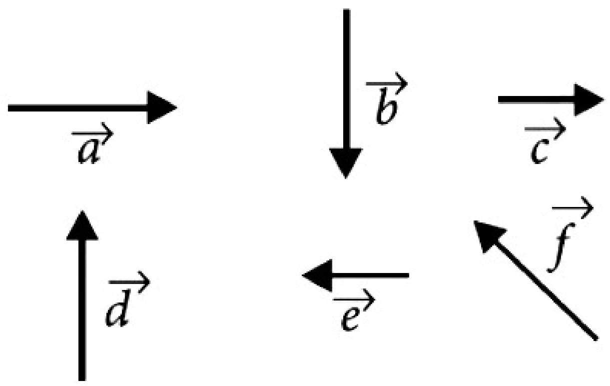
$= 3\hat{i} + 4\hat{j} + 3\hat{j} = 7\hat{i} + 7\hat{j}$

Speed of the particle after 10s $= |\vec{v}|$

$= \sqrt{(7)^2 + (7)^2} = 7\sqrt{2}$ units

Question32

Six vectors, $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true ?



(2010)

Options:

A. $\vec{b} + \vec{c} = \vec{f}$

B. $\vec{d} + \vec{c} = \vec{f}$

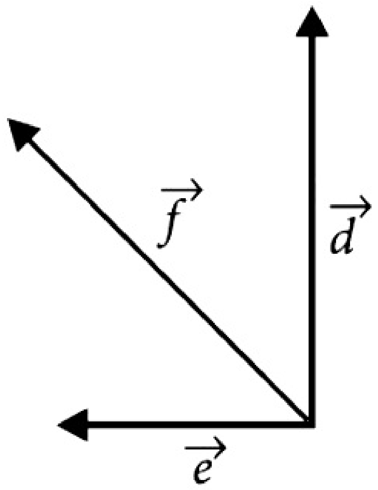
C. $\vec{d} + \vec{e} = \vec{f}$

D. $\vec{b} + \vec{e} = \vec{f}$

Answer: C

Solution:

Solution:



From figure ,$\vec{d} + \vec{e} = \vec{f}$

Question33

The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is
(2010 Mains)

Options:

A. 60°

B. 15°

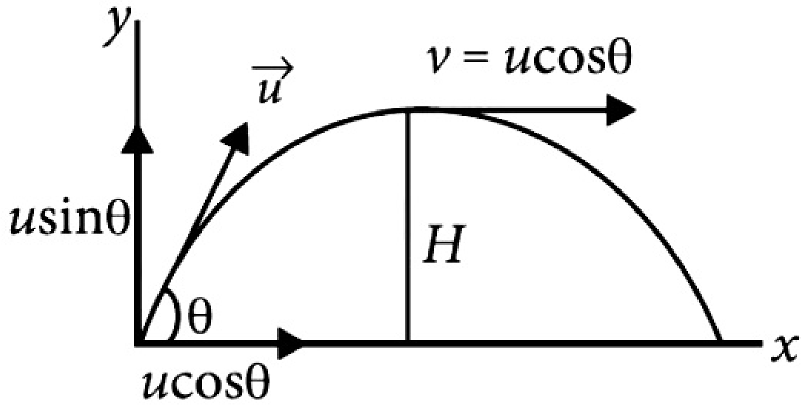
C. 30

D. 45

Answer: A

Solution:

Solution:
Let v be velocity of a projectile at maximum height H.
$v = u\cos\theta$
According to given problem, $v = \frac{u}{2}$
$\therefore \frac{u}{2} = u\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$



Question34

A particle moves in x-y plane according to rule x = asinωt and y = acosωt.The particle follows
(2010 Mains)

Options:

A. an elliptical path

B. a circular path

C. a parabolic path

D. a straight line path inclined equally to x and y-axis

Answer: B

Solution:

Solution:

$x = a\sin\omega t$ or $\frac{x}{a} = \sin\omega t$.........(i)

$y = a\cos\omega t$ or $\frac{y}{a} = \cos\omega t$.........(ii)

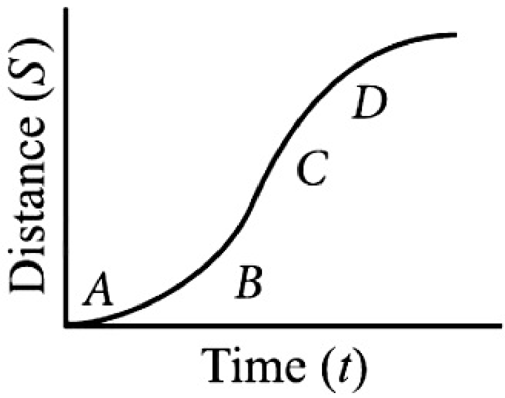
Squaring and adding, we get

$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \quad (\because \cos^2\omega t + \sin^2\omega t = 1)$

$x^2 + y^2 = a^2$

This is the equation of a circle. Hence particle follows a circular path.

Question35



A particle shows distance - time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point (2008)

Options:

A. D

B. A

C. B

D. C

Answer: D

Solution:

Solution:

Because the slope is highest at C,v = $\frac{ds}{dt}$ is maximum.

Question36

A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground the magnitude of the change in its momentum will be

(2008)

Options:

A. $mv\sqrt{2}$

B. zero

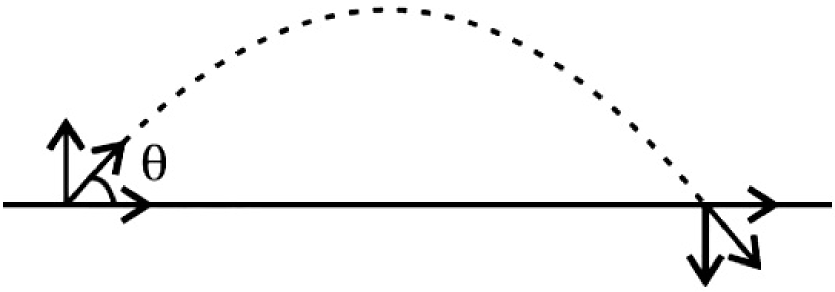
C. 2mv

D. $\frac{mv}{\sqrt{2}}$

Answer: A

Solution:

Solution:



The horizontal momentum does not change. The change in vertical momentum is

$mv\sin\theta - (-mv\sin\theta) = 2mv\frac{1}{\sqrt{2}} = \sqrt{2}mv$

Question37

A particle starting from the origin (0,0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are ($\sqrt{3}$, 3) The path of the particle makes with the x -axis an angle of
(2007)

Options:

A. 45°

B. 60°

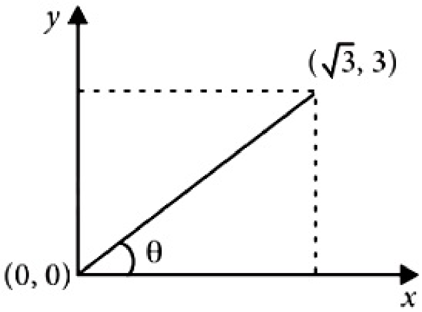
C. 0°

D. 30°.

Answer: B

Solution:

Solution:



Let θ be the angle which the particle makes with an x -axis.
From figure,

$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

or, $\theta = \tan^{-1}(\sqrt{3}) = 60°$

Question38

$\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$, the value of θ is (2007)

Options:

A. 45°

B. 30°

C. 90°

D. 60°

Answer: D

Solution:

Solution:

$|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$

$\therefore AB\sin\theta = \sqrt{3}AB\cos\theta$

or, $\tan\theta = \sqrt{3}$ or, $\theta = \tan^{-1}\sqrt{3} = 60^\circ$

Question39

For angles of projection of a projectile at angle (45° − θ) and (45° + θ), the horizontal range described by the projectile are in the ratio of (2006)

Options:

A. 2: 1

B. 1: 1

C. 2: 3

D. 1: 2

Answer: B

Solution:

Solution:

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

For angle of projection $(45° - \theta)$, the horizontal range is

$\therefore R_1 = \frac{u^2 \sin[2(45° - \theta)]}{g} = \frac{u^2 \sin(90° - 2\theta)}{g}$

$= \frac{u^2 \cos 2\theta}{g}$

For angle of projection left$(45° + \theta)$, the horizontalrange is

$R_2 = \frac{u^2 \sin[2(45° + \theta)]}{g} = \frac{u^2 \sin(90° + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

$\therefore \frac{R_1}{R_2} = \frac{u^2 \cos 2\theta / g}{u^2 \cos 2\theta / g} = \frac{1}{1}$

$\therefore$ The range is the same.

Question40

The vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. The angle between the two vectors is
(2006, 1996, 1991)

Options:

A. 45°

B. 90°

C. 60°

D. 75°

Answer: B

Solution:

Solution:

Let θ be angle between $\vec{A}$ and $\vec{B}$

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

or $(\vec{A} + \vec{B}) . (\vec{A} + \vec{B}) = (\vec{A} - \vec{B}) . (\vec{A} - \vec{B})$

or $\vec{A} . A + \vec{A} . B + \vec{B} . A + \vec{B} . B$

$= \vec{A} . A - \vec{A} . B - \vec{B} . A + \vec{B} . B$

or $4AB \cos\theta = 0$ or $\cos\theta = 0°$ or $\theta = 90°$

Question41

Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other in a time t, where t is
(2005)

Options:

A. $\frac{a}{\sqrt{v^2 + v_1^2}}$

B. $\frac{a}{v + v_1}$

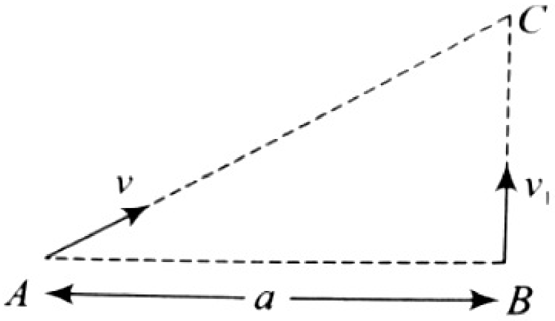
C. $\frac{a}{v - v_1}$

D. $\sqrt{\frac{a^2}{v^2 - v_1^2}}$

Answer: D

Solution:

Solution:



Let two boys meet at point C after time 't' from the starting.
Then AC = vt, BC = v_1t
Using pythagoras theorem
$(AC)^2 = (AB)^2 + (BC)^2$
$\Rightarrow v^2t^2 = a^2 + v_1^2t^2$

By solving we get $\sqrt{\frac{a^2}{v^2 - v_1^2}}$

Question42

A stone tied to the end of a string of 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 seconds, what is the magnitude and direction of acceleration of the stone?
(2005)

Options:

A. $\pi^2 ms^{-2}$ and direction along the radius towards the centre

B. $\pi^2 ms^{-2}$ and direction along the radius away from the centre

C. $\pi^2 ms^{-2}$ and direction along the tangent to the circle

D. $\frac{\pi^2}{4} ms^{-2}$ and direction along the radius towards the centre.

Answer: B

Solution:

Solution:
(a, b) : $a = r\omega^2$; $\omega = 2\pi\upsilon$
22 revolution $= 44\,\text{sec}$
1 revolution $= \frac{44}{22} = 2\,\text{sec}$
$\upsilon = \frac{1}{2} Hz$
$a = r\omega^2 = 1 \times \frac{4\pi^2}{4} = \pi^2 m / s^2$.
Towards the centre, the centripetal acceleration $= -\omega^2 R$
and away from the centre, the centrifugal acceleration is $+\omega^2 R$
∴(a) and (b) are correct as the directions aregiven.

Question43

If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B} \times \vec{A}) . \vec{A}$ is equal to
(2005,1989)

Options:

A. $BA^2 \sin\theta$

B. $BA^2 \cos\theta$

C. $BA^2 \sin\theta \cos\theta$

D. zero.

Answer: D

Solution:

Solution:
Let $\vec{A} \times \vec{B} = \vec{C}$The cross product of $\vec{B}$ and $\vec{A}$ is perpendicularto the plane containing $\vec{A}$ and $\vec{B}$ i.e. perpendicular to $\vec{A}$.
If a dot product of this cross product and $\vec{A}$ is taken, as the cross product is perpendicular to $\vec{A}$, $\vec{C} \times \vec{A} = 0$
Therefore product of $(\vec{B} \times \vec{A}) . \vec{A} \times = 0$.

Question44

If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is
(2005)

Options:

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 1

D. -1

Answer: B

Solution:

$\vec{a} = 2\hat{i} + 3\hat{j} + 8\hat{k}, \vec{b} = 4\hat{j} - 4\hat{i} + \alpha\hat{k}$
$\vec{a} . b = 0$ if $\vec{a} \perp \vec{b}$
$(2\hat{i} + 3\hat{j} + 8\hat{k}) . (-4\hat{j} + 4\hat{i} + \alpha\hat{k} = 0$
or, $-8 + 12 + 8\alpha = 0 \Rightarrow 4 + 8\alpha = 0$
$\Rightarrow \alpha = -\frac{1}{2}$

Question45

If $|\vec{A} \times \vec{B}| = \sqrt{3}\vec{A} . \vec{B}$ then the value of $|\vec{A} + \vec{B}|$ is
(2004)

Options:

A. $(A^2 + B^2 + AB)^{1/2}$

B. $\left(A^2 + B^2 + \frac{AB}{\sqrt{3}}\right)^{1/2}$

C. $A + B$

D. $(A^2 + B^2 + \sqrt{3}AB)^{1/2}$

Answer: A

Solution:

$\left|\vec{A} \times \vec{B}\right| = \sqrt{3}\vec{A} . \vec{B}$
$\left|\vec{A}\right|\left|\vec{B}\right| \sin\theta = \sqrt{3}\left|\vec{A}\right|\left|\vec{B}\right| \cos\theta$
$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$
$\left|\vec{A} + \vec{B}\right| = \sqrt{\left|\vec{A}\right|^2 + \left|\vec{B}\right|^2 + 2\left|\vec{A}\right|\left|B\right| \cos\theta}$
$= (A^2 + B^2 + AB)^{1/2}$

Question46

The vector sum of two forces is perpendicular to their vector differences. In that case, the forces
(2003)

Options:

A. are equal to each other

B. are equal to each other in magnitude

C. are not equal to each other in magnitude

D. cannot be predicted.

Answer: B

Solution:

Solution:
Given : $\left(\vec{F}_1 + \vec{F}_2\right)$ perp left $\left(\vec{F}_1 - \vec{F}_2\right)$
$\therefore \left(\vec{F}_1 + \vec{F}_2\right) . \left(\vec{F}_1 - \vec{F}_2\right) = 0$
$F_1^2 - F_2^2 - \vec{F}_1 . \vec{F}_2 + \vec{F}_2 . \vec{F}_1 = 0 \Rightarrow F_1^2 = F_2^2$
i.e. F_1, F_2 are equal to each other in magnitude.

Question47

A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant quad tangential acceleration. If the velocity of the particle is 80m / s at the end of the second revolution after motion has begun, the tangential acceleration is
(2003)

Options:

A. $40 m/s^2$

B. $640\pi m/s^2$

C. $160\pi m/s^2$

D. $40\pi m/s^2$

Answer: A

Solution:

Solution:
Given: $r = \frac{20}{\pi} m$, $v = 80 m/s$, $\theta = 2\ rev = 4\pi\ rad$
From equation $\omega^2 = \omega_0^2 + 2\alpha\theta (\omega_0 = 0)$
$\omega^2 = 2\alpha\theta \left(\omega = \frac{v}{r} \text{ and } a = r\alpha\right)$
$a = \frac{v^2}{2r\theta} = 40 m/s^2$

Question48

A particle A is dropped from a height and another particle B is projected in horizontal direction with speed of 5m / sec from the same height then correct statement is (2002)

Options:

A. particle A will reach at ground first with respect to particle B

B. particle B will reach at ground first with respect to particle A

C. both particles will reach at ground simultaneously

D. both particles will reach at ground with same speed.

Answer: C

Solution:

Solution:
Time required to reach the ground is dependent on the vertical motion of the particle. Vertical motion of both the particles A and B are exactly same. Although particle B has an initial velocity, but that is in horizontal direction and it has no component in vertical (component of a vector at a direction of $90° = 0$) direction. Hence they will reach the ground simultaneously.

Question49

An object of mass 3kg is at rest. Now a force of $\vec{F} = 6t^2\hat{i} + 4t\hat{j}$ is applied on the object then velocity of object at t = 3s is (2002)

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Options:

A. $18\hat{i} + 3\hat{j}$

B. $18\hat{i} + 6\hat{j}$

C. $3\hat{i} + 18\hat{j}$

D. $18\hat{i} + 4\hat{j}$

Answer: B

Solution:

Mass, $m = 3kg$, force, $F = 6t^2\hat{i} + 4t\hat{j}$
∴ acceleration,

$$a = \frac{F}{m} = \frac{6t^2\hat{i} + 4t\hat{j}}{3} = 2t^2\hat{i} + \frac{4}{3}t\hat{j}$$

Now, $a = \frac{dv}{dt} = 2t^2\hat{i} + \frac{4}{3}t\hat{j}$

$$\therefore dv = \left(2t^2\hat{i} + \frac{4}{3}t\hat{j}\right)dt \therefore v = \int_0^3 \left(2t^2\hat{i} + \frac{4}{3}t\hat{j}\right)dt$$

$$= \frac{2}{3}t^3\hat{i} + \frac{4}{6}t^2\hat{j}\Big|_0^3 = 18\hat{i} + 6\hat{j}$$

Question50

If $\left|\vec{A} + \vec{B}\right| = \left|\vec{A}\right| + \left|\vec{B}\right|$ then angle between A and B will be (2001)

•

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Options:

A. 90°

B. 120°

C. 0°

D. 60°.

Answer: C

Solution:

Solution:
$\left|\vec{A} + \vec{B}\right| = \left|\vec{A}\right| + \left|\vec{B}\right|$ if $\vec{A} || \vec{B}$. $\theta = 0°$

Question51

Two particles having mass M and m are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be
(2001)

Options:

A. $\frac{r}{R}$

B. $\frac{R}{r}$

C. 1

D. $\sqrt{\frac{R}{r}}$

Answer: C

Solution:

Solution:
$\omega = \frac{2\pi}{t}$ As t is same
$\therefore \frac{\omega_1}{\omega_2} = 1$

Question52

The width of river is 1 km. The velocity ofboat is 5 km / hr . The boat covered the width of river in shortest time 15 min. Then the velocity of river stream is
(2000, 1998)

Options:

A. 3 km / hr

B. 4 km / hr

C. $\sqrt{29}$ km / hr

D. $\sqrt{41}$ km / hr

Answer: A

Solution:

$$v_{Resultant} = \frac{1\,km}{1/4\,hr} = 4\,km/hr$$
$$\therefore v_{River} = \sqrt{5^2 - 4^2} = 3\,km/hr$$

Question53

Two projectiles of same mass and with same velocity are thrown at an angle 60° and 30° with the horizontal, then which will remain same (2000)

Options:

A. time of flight

B. range of projectile

C. maximum height acquired

D. all of them.

Answer: B

Solution:

As $\theta_2 = (90 - \theta_1)$

So range of projectile,

$$R_1 = \frac{v_0{}^2 \sin 2\theta}{g} = \frac{v_0{}^2 2 \sin\theta \cos t\theta}{g}$$

$$R_2 = \frac{v_0{}^2 2 \sin(90 - \theta_1)\cos(90 - \theta_1)}{g}$$

$$R_2 = \frac{v_0{}^2 2 \cos\theta_1 \sin\theta_1}{g} = R_1$$

Question54

A man is slipping on a frictionless inclined plane and a bag falls down from the same height. Then the velocity of both is related as

(2000)

Options:

A. $v_B > v_m$

B. $v_B < v_m$

C. $v_B = v_m$

D. v_B and v_m can't be related.

Answer: C

Solution:

Solution:
Vertical acceleration in both the cases is g, whereas horizontal velocity is constant.

Question55

A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km / hr. The centripetal force is (1999)

Options:

A. 1000N

B. 750N

C. 250N

D. 1200N

Answer: A

Solution:

Solution:

$$F_{centripetal} = \frac{mv^2}{R};\ v = \left(36 \times \frac{5}{18}\right) m/s$$

$$F_{centripetal} = \frac{500 \times \left(36 \times \frac{5}{18}\right)^2}{50} = 1000N$$

Question56

A person aiming to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5m / s at an angle of 120° with the direction of flow of water. The speed of water in the stream, is (1999)

Options:

A. 0.25m / s

B. 0.5m / s

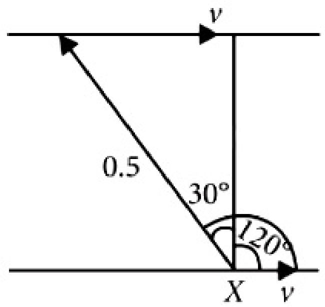
C. 1.0m / s

D. 0.433m / s

Answer: A

Solution:

Solution:



Let v be the velocity of river water. As shown in figure

$\sin 30° = \frac{v}{0.5}$

or, $v = 0.5 \sin 30° = 0.5 \times \left(\frac{1}{2}\right) = 0.25 m / s$

Question57

Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is
(1999)

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Options:

A. $r_1 : r_2$

B. $m_1 : m_2$

C. 1 : 1

D. $m_1 m_2 : r_1 r_2$.

Answer: C

Solution:

Solution:

$t = \frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{1}$

Question58

If a unit vector is represented by $0.5\hat{i} - 0.8\hat{j} + c\hat{k}$ then the value of c is (1999)

Options:

A. $\sqrt{0.01}$

B. $\sqrt{0.11}$

C. 1

D. $\sqrt{0.39}$

Answer: B

Solution:

For a unit vector $\hat{n}$, $|\hat{n}| = 1$
$|0.5\hat{i} - 0.8\hat{j} + c\hat{k}|^2 = 1^2$
$\Rightarrow 0.25 + 0.64 + c^2 = 1$
or $c = \sqrt{0.11}$

Question59

What is the value of linear velocity, if $\vec{r} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{\omega} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?
(1999)

Options:

A. $4\hat{i} - 13\hat{j} + 6\hat{k}$

B. $18\hat{i} + 13\hat{j} - 2\hat{k}$

C. $6\hat{i} + 2\hat{j} - 3\hat{k}$

D. $6\hat{i} + 2\hat{j} + 8\hat{k}$

Answer: B

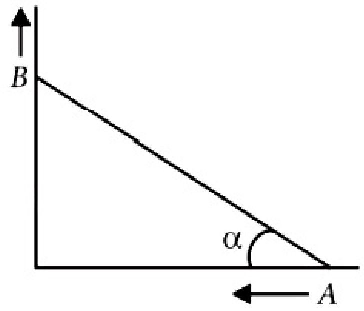
Solution:

Solution:

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -6 & 6 \\ 3 & -4 & 1 \end{vmatrix} = 18\hat{i} + 13\hat{j} - 2\hat{k}$$

Question60

Two particles A and B are connected by a rigid rod AB. The rod slides along perpendicular rails as shown here. The velocity of A to the left is 10m / s . What is the velocity of B when angle α = 60°?



(1998)

Options:

A. 10m / s

B. 9.8m / s

C. 5.8m / s

D. 17.3m / s

Answer: D

Solution:

Solution:
Let us consider the upward velocity of rod B to be vm / s
Now, Velocity of rod A = 10m / s

We know that, Slope of a velocity graph is given by, $\tan\alpha = \frac{v_y}{v_x}$

Here, $v_y = vm/s$ and $v_x = 10m/s$

$\therefore \tan 60^\circ = \frac{v}{10} \Rightarrow v = 10 \times \tan 60^\circ$

$\therefore v = 17.3m/s$

Question61

A ball of mass 0.25 kg attached to the end of a string of length 1.96m is moving in a horizontal circle. The string will break if the tension is more than 25N. What is the maximum speed with which the ball can be moved?
(1998)

Options:

A. 5m / s

B. 3m / s

C. 14m / s

D. 3.92m / s

Answer: C

Solution:

Solution:

$\frac{mv^2}{r} = 25;$

$v = \sqrt{\frac{25 \times 1.96}{0.25}} = 14m / s$

Question62

Identify the vector quantity among the following. (1997)

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Options:

A. distance

B. angular momentum

C. heat

D. energy.

Answer: B

Solution:

Solution:
since the angular momentum has both magnitude and direction, it is a vector quantity.

Question63

A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad / s. What is its linear velocity at any point on circular path? (1996)

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Options:

A. 20m / s

B. $\sqrt{2}$m / s

C. 10m / s

D. 2m / s

Answer: D

Solution:

Solution:
Radius of circle (r) = 20 cm = 0.2m and angular velocity (ω) = 10 rad / s
linear velocity (v) = $r\omega = 0.2 \times 10 = 2$m / s

Question64

The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is (1995)

Options:

A. directed towards the origin

B. directed away from the origin

C. parallel to the position vector

D. perpendicular to the position vector.

Answer: D

Solution:

Solution:
position vector of particle
$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$
velocity vector
$\vec{v} = \frac{d\vec{r}}{dt} = (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$
$= \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j}]$
$\vec{v}.\vec{r} = \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j}].[(a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}] = 0$
Therefore velocity vector is perpendicular to the displacement vector.

Question65

The angular speed of a flywheel making 120 revolutions/minute is (1995)

Options:

A. 4π rad / s

B. $4\pi^2$ rad / s

C. π rad / s

D. 2π rad / s

Answer: A

Solution:

Solution:
Number of revolutions per minute (n) = 120
Therefore angular speed (ω) = $\frac{2\pi n}{60} = \frac{2\pi \times 120}{60} = 4\pi$ rad / s

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Question66

The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be
(1994)

Options:

A. 90°

B. 180°

C. zero

D. 45°.

Answer: A

Solution:

Solution:
$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$$\cos\theta = \frac{\vec{A}.B}{|\vec{A}||\vec{B}|}$$

$$= \frac{(3\hat{i} + 4\hat{j} + 5\hat{k}).(3\hat{i} + 4\hat{j} - 5\hat{k})}{[\sqrt{(3)^2 + (4)^2 + (5)^2}] \times [\sqrt{(3)^2 + (4)^2 + (5)^2}]}$$

$$= \frac{9 + 16 - 25}{50} = 0 \text{ or } \theta = 90°$$

Question67

A boat is sent across a river with a velocity of $8\,km\,h^{-1}$. If the resultant velocity of boat is $10\,km\,h^{-1}$, then velocity of river is
(1994, 1993)

Options:

A. $12.8\,km\,h^{-1}$

B. $6\,km\,h^{-1}$

C. $8\,km\,h^{-1}$

D. $10\,km\,h^{-1}$

Answer: B

Solution:

Solution:
Let the velocity of river be v_R and velocity of boat is v_B
$\therefore$ Resultant velocity $= \sqrt{{v_B}^2 + {v_R}^2 + 2v_Bv_R\cos\theta}$
$(10) = \sqrt{{v_B}^2 + {v_R}^2 + 2v_Bv_R\cos 90°}$
$(10) = \sqrt{(8)^2 + {v_R}^2}$ or $(10)^2 = (8)^2 + {v_R}^2$
${v_R}^2 = 100 - 64$ or $v_R = 6\,km\,hr$

Question68

If a body A of mass M is thrown with velocity v at an angle of 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal, the ratio of horizontal range of A to B will be
(1992, 1990)

Options:

A. 1: 3

B. 1: 1

C. $1 : \sqrt{3}$

D. $\sqrt{3} : 1$.

Answer: B

Solution:

For the given velocity of projection u, the horizontal range is the same for the angle of projection θ and $90° - \theta$

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

∴ For body A, $R_A = \frac{u^2 \sin(2 \times 30°)}{g} = \frac{u^2 \sin 60°}{g}$

For bodyB, $R_B = \frac{u^2 \sin(2 \times 60°)}{g}$

$R_B = \frac{u^2 \sin 120°}{g} = \frac{u^2 \sin(180° - 60°)}{g} = \frac{u^2 \sin 60°}{g}$

The range is the same whether the angle is theta or $90° - \theta$

∴ The ratio of ranges is 1: 1

Question69

The resultant of $\vec{A} \times 0$ will be equal to (1992)

Options:

A. zero

B. A

C. zero vector

D. unit vector.

Answer: C

Solution:

Solution:

The cross product $\vec{A} \times \vec{B}$ is a vector, with its direction perpendicular to both $\vec{A}$ and $\vec{B}$. $\vec{A} \times \vec{B}$ is area. If side B is zero, area is zero.$\vec{A} \times 0$ is a zero vector.

If in case 0 is a scalar, then also the product is zero. But a scalar × a vector is also a vector.Hence one gets a zero vector in any case.

Question70

An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is (1990)

Options:

A. 1600 ms^{-2}

B. 47.4 ms^{-2}

C. 23.7 ms^{-2}

D. 50.55 ms^{-2}

Answer: B

Solution:

Frequency of rotation $\upsilon = 120\,rpm$
$= 2\,rps$
length of blade $r = 30cm = 0.3m$
Centripetal acceleration $a = \omega^2 r = (2\pi\upsilon)^2 r$
$= 4\pi^2\upsilon^2 r = 4\pi^2(2)^2(0.3) \; = 47.4\ ms^{-2}$

Question71

The maximum range of a gun of horizontal terrain is 16km. If $g = 10\ ms^{-2}$, then muzzle velocity of a shell must be
(1990)

Options:

A. 160 ms^{-1}

B. $200\sqrt{2}\ ms^{-1}$

C. 400 ms^{-1}

D. 800 ms^{-1}

Answer: C

Solution:

Solution:

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$
For maximum horizontal range $\theta = 45°$
or $R_m = \frac{u^2}{g}$
where u be muzzle velocity of a shell
$\therefore (1600m) = \frac{u^2}{(10\ ms^{-2})^2}$ or $u = 400\ ms^{-1}$

Question72

A bus is moving on a straight road towards north with a uniform speed of 50 km / hour then it turns left through 90°. If the speed remains unchanged after turning, the increase in the velocity of bus in the turning process is
(1989)

Options:

A. 70.7 km / hour along south-west direction

B. zero

C. 50 km / hour along west

D. 70.7 km / hour along north-west direction

Answer: A

Solution:

Solution:
$v_1 = 50\,\text{km / hr}$ due north
$v_2 = 50\,\text{km / hr}$ due west
$-v_1 = 50\,\text{km / hr}$ due south
Magnitude of change in velocity = $\vec{v}_2 - \vec{v}_1 \big| = \big| \vec{v}_2 + \left(-\vec{v}_1\right) \big| = \sqrt{v_2^{\,2} + (-v_1)^2}$
$= \sqrt{(50)^2 + (50)^2} = 70.7\,\text{km / hr}$
$\vec{v} = 70.7\,\text{km / hr}$ along south-west direction

Question73

The magnitude of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 3,4 and 5 units respectively. If $\vec{A} + \vec{B} = \vec{C}$ the angle between $\vec{A}$ and $\vec{B}$ is (1988)

Options:

A. $\frac{\pi}{2}$

B. $\cos^{-1}(0.6)$

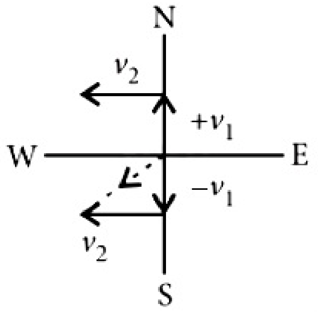
C. $\tan^{-1}\left(\frac{7}{5}\right)$

D. $\frac{\pi}{4}$

Answer: A

Solution:

Solution:



Let θ be angle between $\vec{A}$ and $\vec{B}$
Given : $A = |\vec{A}| = 3$ units
$B = |\vec{B}| = 4$ units
$C = |\vec{C}| = 5$ units
$\vec{A} + \vec{B} = \vec{C}$
$\therefore (\vec{A} + \vec{B}) . (\vec{A} + \vec{B}) = \vec{C} . \vec{C}$
$\vec{A} . \vec{A} + \vec{A} . \vec{B} + \vec{B} . \vec{A} + \vec{B} . \vec{B} = \vec{C} . \vec{C}$
$A^2 + 2AB\cos\theta + B^2 = C^2$
$9 + 2AB\cos\theta + 16 = 25$ or $2AB\cos\theta = 0$
or $\cos\theta = 0 \therefore \theta = 90°$

Question74

A train of 150 metre length is going towards north direction at a speed of 10m / s. A parrot flies at the speed of 5m / s towards south direction parallel to the railways track. The time taken by the parrot to cross the train is
(1988)

Options:

A. 12s

B. 8s

C. 15s

D. 10s

Answer: D

Solution:

Choose the positive direction of x -axis to be from south to north. Then
velocity of train $v_T = +10ms^{-1}$
velocity of parrot $v_P = -5ms^{-1}$
Relative velocity of parrot with respect to train $= v_P - v_T = (-5ms^{-1}) - (+10ms^{-1})$
$= -15ms^{-1}$
i.e. parrot appears to move with a speed of $15ms^{-1}$ from north to south
$\therefore$ Time taken by parrot to cross the train $= \dfrac{150m}{15ms^{-1}} = 10s$
